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Enrico Santamato* [a], Bruno Piccirillo [a], Antonio Setaro [a] & Angela Vella [a]

[a] Dipartimento di Scienze Fisiche, INFM, Complesso di Monte S. Angelo, via Cintia, Napoli

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THE ORBITAL ANGULAR MOMENTUM OF LIGHT: A NEW SOURCE OF OPTICAL TORQUE IN LIQUID CRYSTALS

Enrico Santamato, Bruno Piccirillo, Antonio Setaro, and Angela Vella*
INFM, Dipartimento di Scienze Fisiche, Complesso di Monte S. Angelo, via Cintia, 80126 Napoli

We present a study on the flux of angular momentum from a monochromatic optical field into a nematic liquid crystal. In particular, we investigate, starting form first principles, the role played by the orbital part of the angular momentum of light in reorienting the sample. We demonstrated that, when the fluid motion is neglected, the photon spin couples with the director ***n*** *directly, while the photon orbital angular momentum couples with the gradients of* ***n****. This work was motivated by the recent experiments where the transfer of the orbital part of the angular momentum carried by a laser beam into a nematic film has been claimed. A possible approach to model experimental results is also suggested.*

Keywords: angular momentum of light; liquid crystals; nonlinear optics

1. INTRODUCTION

Recent experiments have demonstrated that the optical reorientation of liquid crystals can be dramatically affected when the shape of the incident laser beam is made elliptical rather then circular [1–6]. Multistability, out-of-incidence-plane reorientation and nonlinear oscillations have been observed even in the simple geometry of the Optical Fréedericksz Transition (OFT), i.e., using a linearly polarized laser beam normally incident onto a homeotropically aligned nematic film. All these phenomena are absent when the laser beam cross section is circular. When a circularly polarized elliptically shaped laser beam is used, the laser-induced uniform rotation of the molecular director observed long time ago with laser beams having circular cross-section [7,8] becomes chaotic and on-off intermittency develops spontaneously [6]. All these phenomena were ascribed to

*Corresponding author. Tel.: +39 081 676359, Fax: +39 081 676346, E-mail: enrico.santamato@na.infn.it

the interplay between the orbital and the spin angular momentum of light acting on the liquid crystal [2]. The sensitivity of liquid crystals to the spin part of the angular momentum of light is well known [7,8] and light-driven molecular motors are based on it [9]. That liquid crystals may be sensitive to the orbital part of the angular momentum of light is a recent achievement and it is based on the observation that liquid crystals do suffer a torque along the laser beam axis even when the light is unpolarized, provided that the beam cross section is made elliptical [1,2]. Nonetheless, the connection between the optical torque acting on the liquid crystal sample and the orbital angular momentum carried by the incident light beam is still unclear and the experimental observations are indirect, because measuring the orbital angular momentum of light is very difficult in practical cases. Roughly speaking, we may say that the spin part of a light beam acts on the director $\boldsymbol{n}$ directly, while the orbital part acts on the gradients of $\boldsymbol{n}$ [5]. This statement can be easily understood looking at Figure 1, where a

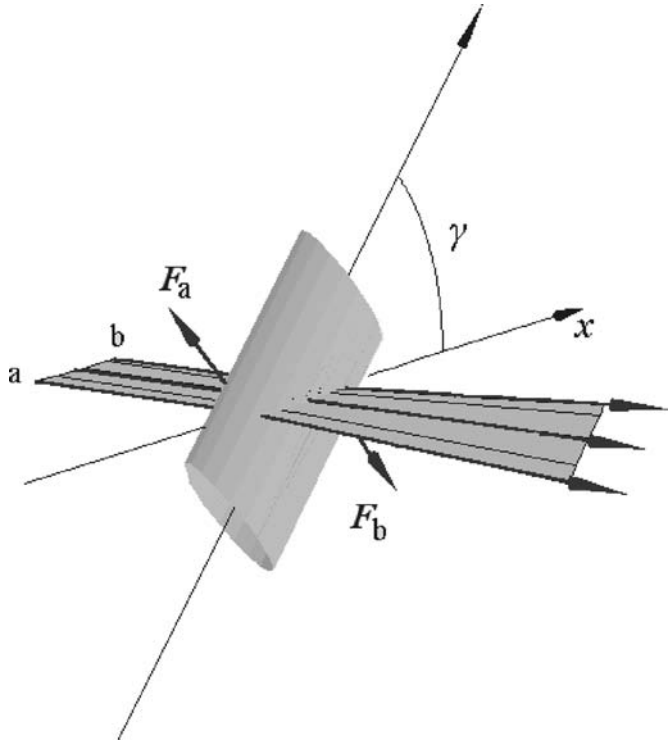

FIGURE 1 The reoriented liquid crystal can be considered as a cylindrical lens. Refraction of the elongated incident beam by the lens results in a couple of forces F_a and F_b acting on it.

light beam having a profile strongly elongated in the horizontal direction is incident onto a cylindrical lens with cylindrical axis tilted at angle γ. The rays passing through the lens are bent so that the photon recoil produces a couple of forces $\boldsymbol{F_a}$ and $\boldsymbol{F_b}$ and, hence, a torque acting on the lens itself. This torque does not depend on the polarization of the light beam, but on the curvature of the photon trajectories. We may say, therefore, that the torque acting on the lens is originated by the orbital angular momentum of light. A gradient of the director $\boldsymbol{n}$ produces a corresponding gradient of the refractive index of the material. If the index profile in the liquid crystal becomes roughly equivalent to the lens in Figure 1, an optical torque arises that has nothing to do with the light polarization. We are forced to ascribe this torque to the orbital angular momentum extracted from the incident radiation by the medium.

The aim of this paper is to derive from first principles the connection between the angular momentum carried by a light beam and the forces and torques acting in the material, and to present some useful equations that could be used to model the optical reorientation when the orbital angular momentum of light is involved. We need a general approach where all space coordinates are retained, because the plane-wave approximation is useless. Plane waves, in fact, carry no orbital angular momentum and transverse gradients of $\boldsymbol{n}$ are needed, as shown by the example in Figure 1. The paper is organized as follows. In Section 2, the fundamental equations of hydrodynamics of a liquid crystal in an optical field are presented in terms of the flux densities of linear and angular momentum of the electromagnetic field. We present also a way to split the total angular momentum of the monochromatic electromagnetic field into an orbital and an intrinsic part which are separately conserved in isotropic and homogeneous media. In Section 3, a possible way to handle the interaction of liquid crystals with the orbital angular momentum of light in practical situations is envisaged. In Section 4 our conclusions are drawn.

2. THE OPTICAL ANGULAR MOMENTUM FLUX

We start from the fundamental equations of hydrodynamics of liquid crystals.

$$\dot{\boldsymbol{p}} = \boldsymbol{f} = -\mathrm{div}\,\hat{\boldsymbol{\sigma}} \tag{1a}$$

$$\dot{l} = \boldsymbol{r} \times \boldsymbol{f} = -\mathrm{div}\,\hat{L} + \boldsymbol{\omega} \tag{1b}$$

$$\dot{\boldsymbol{s}} = \boldsymbol{\tau} = -\mathrm{div}\,\hat{S} - \boldsymbol{\omega} \tag{1c}$$

where the dot means the total time derivative and $\boldsymbol{p}, \boldsymbol{l} = \boldsymbol{r} \times \boldsymbol{p}$, and $\boldsymbol{s}$ are the linear momentum, the orbital (fluid center-of-mass) angular momentum and the intrinsic angular momentum for unit volume of the fluid. The quantities $\boldsymbol{p}$

and $\boldsymbol{l}$ are the same as in ordinary fluids, while $\boldsymbol{s} = I(\boldsymbol{n} \times \dot{\boldsymbol{n}})$ is due to the internal orientational degrees of freedom characteristic of liquid crystals (I is a measure of the molecular momentum of inertia). In Eqs. (1), $\boldsymbol{f}$ and $\boldsymbol{\tau}$ are the total force and torque per unit volume. Elastic, optical and viscous terms contribute to $\boldsymbol{f}$ and $\boldsymbol{\tau}$. The quantities $\hat{\sigma}$, $\hat{L}$, and $\hat{S}$ are tensors that represent the flux densities of the linear momentum (i.e., the stress tensor), the orbital angular momentum and the intrinsic angular momentum entering the surface of the considered volume element dV. The notation $\operatorname{div}\hat{T}$ for a generic tensor $\hat{T}$ means the divergence calculated as $(\operatorname{div}\hat{T})_\alpha = \partial_\rho T_{\rho\alpha}$. Finally, $\boldsymbol{\omega}$ is the vector dual of the antisymmetric part of the stress tensor $\hat{\sigma}$, i.e., $\omega_\alpha = \varepsilon_{\alpha\beta\gamma}\sigma_{\beta\gamma}$. When Eq. (1a) is integrated over the volume V of the sample, it states that the time change of the total linear momentum of the system comes out from external forces acting through its surface, as stated by Newton's law. Equation (1b) is a consequence of the first one and of the definition of $\hat{L}$ as $L_{\rho\alpha} = \varepsilon_{\alpha\beta\gamma}x_\beta\sigma_{\rho\gamma}$. When integrated over the volume V, the term on the right does not reduce to a surface integral because of the presence of the internal torque $\boldsymbol{\omega}$. Equation (1c) is written out so to obtain $\dot{\boldsymbol{l}} + \dot{\boldsymbol{s}} = \operatorname{div}\hat{J}$ where $\hat{J} = \hat{L} + \hat{S}$ is the flux density of the total angular momentum crossing the surface of dV. When integrated over V, the last relation shows that the total (orbital + intrinsic) angular momentum of the material comes out from external forces only, as stated by Newton's law. Elastic, optical and viscous terms contribute to $\hat{\sigma}$, $\boldsymbol{\omega}$, and $\hat{L}$, while $\hat{S}$ has only the elastic and the optical contributions, because strong anchoring assures the absence of friction on $\boldsymbol{n}$ at the sample walls. As a consequence, the viscous torque acting on the director is given by $\boldsymbol{\tau}^v = -\boldsymbol{\omega}^v$, i.e. by the antisymmetric part of the viscous stress tensor $\hat{\sigma}^v$ (see Ref. [10], Sec. 5.1.3.3; the change of sign is due to the fact that we are considering here the flux entering the volume dV). The elastic contribution $\hat{\sigma}^e$ to the stress tensor $\hat{\sigma}$ is known in the hydrodynamics of liquid crystals as the Ericksen stress tensor and it is usually derived from a suitable elastic free energy functional $\Phi^e = \int_V F^e dV$. The explicit expression of F^e as a function of $\boldsymbol{n}$ and its space derivatives for nematics is found in standard textbooks on the physics of liquid crystals and it contains three elastic constants k_{11}, k_{22}, k_{33} associated to splay, twist and bend deformations respectively [10]. The connection between F^e and the Ericksen stress tensor $\hat{\sigma}^e$ is given by $\sigma^e_{\alpha\beta} = p^e_{\alpha\gamma}\partial_\beta n_\gamma - \delta_{\alpha\beta}F^e$, where $p^e_{\alpha\gamma} = \frac{\partial F^e}{\partial(\partial_\alpha n_\gamma)}$ is the so-called "molecular field". The elastic contribution $\hat{L}^e$ to the orbital angular momentum flux density $\hat{L}$ is $L^e_{\rho\alpha} = \varepsilon_{\alpha\beta\gamma}x_\beta\sigma^e_{\rho\gamma}$. The Ericksen stress tensor $\hat{\sigma}^e$ is not symmetric, in general, hence $\boldsymbol{\omega}^e \neq 0$. The elastic contribution $\boldsymbol{\tau}^e$ to the torque $\boldsymbol{\tau}$ acting on $\boldsymbol{n}$ can also be derived from F^e as $\boldsymbol{\tau}^e = \boldsymbol{n} \times \boldsymbol{h}^e$, where $\boldsymbol{h}^e = \operatorname{div}\hat{p}^e - \frac{\partial F^e}{\partial \boldsymbol{n}}$. Equation (1c) results from the rotational invariance of F^e (Noether's theorem), provided that the flux density $\hat{S}$ is defined as $S^e_{\alpha\beta} = \varepsilon_{\beta\mu\nu}n_\mu p^e_{\alpha\nu}$.

The optical contribution can be calculated in an analogous way by adding to the free energy the electromagnetic term $\Phi^{em} = \int_V F^{em} dV$, with F^{em}

given by

$$F^{em} = \frac{1}{16\pi}(\boldsymbol{B}^* \cdot \boldsymbol{H} - \boldsymbol{D}^* \cdot \boldsymbol{E}), \tag{2}$$

where at the optical frequency ω we have $\boldsymbol{B} = \boldsymbol{H} = -\frac{i}{k_0}\mathrm{rot}\,\boldsymbol{E}$, $k_0 = \frac{\omega}{c}$, and $\boldsymbol{D} = \hat{\varepsilon}\boldsymbol{E}$, with dielectric tensor given by $\varepsilon_{\alpha\beta} = \varepsilon_o\delta_{\alpha\beta} + \varepsilon_a n_\alpha n_\beta$ and constant ε_o and ε_a. It is easily verified that if we consider $\boldsymbol{E}$ and $\boldsymbol{E}^*$ as field variables in F^e_m, the resulting field equations are Maxwell's equations rotrot $\boldsymbol{E} = k_0^2\hat{\varepsilon}\boldsymbol{E}$.

The optical contribution $\hat{\boldsymbol{\sigma}}^{em}$ to the stress tensor is given by $\sigma^{em}_{\alpha\beta} = (p^{em}_{\alpha\gamma}\partial_\beta E^*_\gamma + \mathrm{c.c.}) - \delta_{\alpha\beta}F^{em}$, with $p^{em}_{\alpha\gamma} = \frac{\partial F^e}{\partial(\partial_\alpha E^*_\gamma)}$. The tensor $\hat{\boldsymbol{\sigma}}^{em}$ is not symmetric, hence $\boldsymbol{\omega}^{em} \neq 0$. Taking the divergence of $\hat{\boldsymbol{\sigma}}^{em}$ we obtain the optical contribution $\boldsymbol{f}^{em}$ to the force $\boldsymbol{f}$ per unit volume as $f^{em}_\alpha = -\partial_\rho\sigma^{em}_{\rho\alpha} = -\frac{1}{16\pi}E_\gamma E^*_\rho\partial_\alpha\varepsilon_{\gamma\rho}$, which is a well known result in birefringent media [11]. The optical contribution $\boldsymbol{\tau}^{em}$ to the torque acting on $\boldsymbol{n}$ is given by $\boldsymbol{\tau}^{em} = -\boldsymbol{n}\times\frac{\partial F^{em}}{\partial \boldsymbol{n}} = \frac{1}{16\pi}(\boldsymbol{D}^*\times\boldsymbol{E} + \mathrm{c.c.})$, which is a well known result, and the optical contribution $\hat{S}^{em}$ to the flux density of the electromagnetic intrinsic angular momentum is $S^{em}_{\rho\alpha} = \varepsilon_{\alpha\mu\nu}(p^{em}_{\rho\mu}E^*_\nu + \mathrm{c.c.})$. The flux density $\hat{L}^{em}$ of the electromagnetic orbital angular momentum is defined as $L^{em}_{\rho\alpha} = \varepsilon_{\alpha\beta\gamma}x_\beta\sigma^{em}_{\rho\gamma}$. When a light beam is considered propagating along the z-axis, the flux densities of the z-component of the orbital and intrinsic angular momentum across a plane transverse to the beam are L^{em}_{zz} and S^{em}_{zz}, respectively. It is worth noting that L^{em}_{zz} and S^{em}_{zz} calculated from F^{em} given by Eq. (2) are the same as the ones suggested recently after a long calculation starting from Maxwell's stress tensor in vacuum [12]. It is also noticeable that these flux densities retain the same form in vacuum and in a birefringent medium. We report, finally, a useful relationship coming from the invariance of $F = F^e + F^{em}$ with respect to infinitesimal rotations of the molecular center of mass coordinates $\boldsymbol{r}$:

$$\mathrm{div}\,\hat{L}^e + \mathrm{div}\,\hat{L}^{em} - \boldsymbol{\omega}^e - \boldsymbol{\omega}^{em} = (h^e_\rho + h^{em}_\rho)(\boldsymbol{r}\times\nabla)n_\rho \tag{3}$$

As it concerns the viscous contributions, they are the same as reported in standard textbooks on liquid crystals [9], because no entropy production is associated to the optical field in a transparent medium.

Now we pass to discuss Eqs. (1). It is well known that the stress tensor $\hat{\boldsymbol{\sigma}}$ is not unique. For example, we may add to $\sigma_{\alpha\beta}$ a term as $\partial_\rho f_{\rho\alpha\beta}$ with $f_{\rho\alpha\beta} = -f_{\alpha\rho\beta}$ without affecting Eqs. (1) provided that we change also $\hat{L}$ and $\hat{S}$ according to $L_{\rho\alpha} \to L_{\rho\alpha} + \varepsilon_{\alpha\beta\gamma}x_\beta\partial_\mu f_{\mu\rho\gamma}$ and $S_{\rho\alpha} \to S_{\rho\alpha} - \varepsilon_{\alpha\beta\gamma}f_{\rho\beta\gamma}$. We notice that also the density $\hat{J}$ of the total angular momentum changes in this transformation, only its divergence being invariant. We may uniquely determine the function $f_{\alpha\beta\gamma}$ so to have arbitrary values assigned to $S_{\alpha\beta}$. We may, for example, choose $f_{\alpha\beta\gamma}$ so to have $S_{\alpha\beta} = 0$. When this choice is made,

Eq. (1c) reduces to $\dot{\boldsymbol{s}} = \boldsymbol{\omega}$, showing that the total stress tensor $\hat{\boldsymbol{\sigma}}$ cannot be symmetric, whenever $\dot{\boldsymbol{s}} \neq 0$. In the case of liquid crystals, the molecular momentum of inertia I is very small and we may assume $\dot{\boldsymbol{s}} = I(\boldsymbol{n} \times \ddot{\boldsymbol{n}}) = \boldsymbol{\tau} = 0$, i.e., we may assume that the elastic, viscous and optical torques per unit volume exactly balance each other. Within this assumption, when the stress tensor is chosen so to have $\hat{S} = 0$, we have $\boldsymbol{\omega} = 0$ and the stress tensor $\hat{\boldsymbol{\sigma}}$ becomes symmetric. This way to obtain a symmetric stress tensor in liquid crystals was proposed long time ago [13], but it has the drawback of being based on the dynamical constraint $\dot{\boldsymbol{s}} = 0$. When this symmetrization procedure is generalized to include the electromagnetic stress tensor too, the final result is that whenever $\dot{\boldsymbol{s}} = 0$ in the material, we may take $\hat{\boldsymbol{\sigma}}^{em}$ as the symmetric part of Maxwell's stress tensor, a well known result in the case of solid crystals, which have no internal orientational degree of freedom (see Ref. [14], Sec. 16). On the other hand, the Ericksen stress tensor $\hat{\boldsymbol{\sigma}}^{e}$ and the electromagnetic stress tensor $\hat{\boldsymbol{\sigma}}^{em}$ derived from the free energy Φ are not symmetric, so that Eqs. (1b) and (1c) are coupled by the internal torque $\boldsymbol{\omega}$, that, in general, cannot be suppressed. The lack of symmetry of $\hat{\boldsymbol{\sigma}}^{e}$ and $\hat{\boldsymbol{\sigma}}^{em}$ is due ultimately to the elastic and the optical anisotropy of the material, respectively. Unfortunately, even when all the elastic constants are set to a common value K, the Ericksen tensor $\hat{\boldsymbol{\sigma}}^{e}$ still remains non-symmetric. Similarly, $\hat{\boldsymbol{\sigma}}^{em}$ is not symmetric also when homogeneous and isotropic media as vacuum are envisaged. We may look, however, for stress tensors $\hat{\boldsymbol{\sigma}}^{e}$ and $\hat{\boldsymbol{\sigma}}^{em}$ becoming symmetric in the limit of isotropic materials and leaving Eqs. (1) unchanged. A possible choice is

$$\sigma^{e}_{\alpha\beta} = k_{22}\left(\partial_\alpha n_\gamma \partial_\beta n_\gamma - \frac{1}{2}\delta_{\alpha\beta}\partial_\gamma n_\rho \partial_\gamma n_\rho\right) + \delta k_{11}\left[\operatorname{div}\boldsymbol{n}\,\partial_\beta n_\alpha + \frac{1}{2}\delta_{\alpha\beta}A^2\right] - \delta k_{33}\left[(n_\gamma B_\alpha - n_\alpha B_\gamma)\partial_\beta n_\gamma + \frac{1}{2}\delta_{\alpha\beta}B^2\right] \tag{4}$$

$$\sigma^{em}_{\alpha\beta} = \frac{1}{16\pi k_0^2}[(\partial_\alpha E_\gamma \partial_\beta E^*_\gamma - \operatorname{div}\boldsymbol{E}\partial_\beta E^*_\alpha + c.c.) + \delta_{\alpha\beta}(\partial_\gamma E_\rho \partial_\gamma E^*_\rho - \operatorname{div}\boldsymbol{E}\operatorname{div}\boldsymbol{E}^* - k_0^2 \varepsilon_{\gamma\rho} E_\gamma E^*_\rho)] \tag{5}$$

where we posed $A = \boldsymbol{n}\cdot\operatorname{rot}\boldsymbol{n}$, $\boldsymbol{B} = \boldsymbol{n}\times\operatorname{rot}\boldsymbol{n}$, $\delta k_{11} = 1 - \frac{k_{11}}{k_{22}}$, $\delta k_{33} = 1 - \frac{k_{33}}{k_{22}}$. The electromagnetic stress tensor in Eq. (5) becomes symmetric in isotropic homogeneous media, because in this case we have $\operatorname{div}\boldsymbol{E} = 0$. From Eq. (5), in fact, we obtain

$$\boldsymbol{\omega}^{em} = -\frac{1}{16\pi k_0^2}\operatorname{rot}\boldsymbol{E}^*\operatorname{div}\boldsymbol{E} + c.c. \tag{6}$$

The flux densities $\hat{L}^e$ and $\hat{L}^{em}$ of the orbital angular momentum are obtained from Eqs. (4) and (5) by the formal substitutions $\partial_\beta \rightarrow (\boldsymbol{r} \times \nabla)_\beta$ and $\delta_{\alpha\beta} \rightarrow \varepsilon_{\alpha\beta\rho} x_\rho$, respectively. The flux densities $\hat{S}^e$ and $\hat{S}^{em}$ of the elastic and optical intrinsic angular momentum are given by

$$S^e_{\alpha\beta} = -k_{22} \varepsilon_{\beta\mu\nu} n_\mu \partial_\alpha n_\nu \tag{7}$$

$$S^{em}_{\alpha\beta} = -\frac{1}{16\pi k_0^2} (\varepsilon_{\beta\mu\nu} E^*_\mu \partial_\alpha E_\nu - \varepsilon_{\alpha\beta\mu} E^*_\mu \mathrm{div}\, \boldsymbol{E}) + \mathrm{c.c.} \tag{8}$$

respectively. A very useful property of the electromagnetic flux densities $\hat{L}^{em}$ and $\hat{S}^{em}$ defined by Eqs. (7) and (8) is that they are both divergence free in isotropic and homogeneous media. In this way, the total electromagnetic angular momentum flux $\hat{J}^{em} = \hat{L}^{em} + \hat{S}^{em}$ is split in the orbital and intrinsic parts so to have the two parts conserved separately in homogeneous and isotropic media. Moreover, when L^{em}_{zz} and S^{em}_{zz} are evaluated from Eqs. (5) and (8), and a collimated beam is considered propagating in vacuum along the z-direction, we obtain the well known expressions of the flux densities of the orbital and spin angular momentum of light in the paraxial optics approximation [12,15]

$$L^{em}_{zz} \cong \frac{i}{16\pi k_0} [E_x (x\partial_y - y\partial_x) E^*_x + E_y (x\partial_y - y\partial_x) E^*_y - c.c] \tag{9}$$

$$S^{em}_{zz} \cong \frac{i}{8\pi k_0} (E_x E^*_y - E_y E^*_x). \tag{10}$$

3. TOWARDS A MODEL

In this section we use the relationships obtained above to suggest a way to model the recent experiments made on liquid crystals where the orbital angular momentum of light is involved [1–6]. It is obvious that drastic simplifications are mandatory in order to obtain a workable problem. As said above, we cannot use the plane wave approximation, but the paraxial optics is adequate when laser beam are used. A further approximation is completely neglecting the motion of the fluid, assuming its velocity $\boldsymbol{v} = 0$. In this way the only degree of freedom we are left with is the director $\boldsymbol{n}$. We may also neglect the momentum of inertia I and assume the local balance of the torques, $\boldsymbol{\tau} = \boldsymbol{\tau}^e + \boldsymbol{\tau}^{em} + \boldsymbol{\tau}^v = 0$, as usual in liquid crystals. We can no longer use Eqs. (1a) and (1c) because they are related to the

fluid motion, but we can use instead Eq. (3) in its integrated form

$$\oint_{\partial V} d\sigma \mathbf{u}\cdot\hat{L}^{e}+\oint_{\partial V} d\sigma \mathbf{u}\cdot\hat{L}^{em}=-\int_{V} dV h_{\rho}^{v}(\mathbf{r}\times\nabla)n_{\rho}+\int_{V} dV(\boldsymbol{\omega}^{e}+\boldsymbol{\omega}^{em})$$
$$=-\gamma_{1}\int_{V} dV\partial_{t}n_{\rho}(\mathbf{r}\times\nabla)n_{\rho}+\int_{V} dV(\boldsymbol{\omega}^{e}+\boldsymbol{\omega}^{em}), \tag{11}$$

where $\boldsymbol{u}$ is the exterior normal to the surface ∂V enclosing the sample and we used the fact that when I is neglected we have $\boldsymbol{h^{e}}+\boldsymbol{h^{em}}=-\boldsymbol{h}^{v}=-\gamma_{1}\partial_{t}\boldsymbol{n}$, where γ_1 is a viscosity coefficient. Integration of Eq. (1c) over the volume V of the sample yields

$$\oint_{\partial V} d\sigma \mathbf{u}\cdot\hat{S}^{e}+\oint_{\partial V} d\sigma \mathbf{u}\cdot\hat{S}^{em}=\int_{V} dV\boldsymbol{\tau}^{v}-\int_{V} dV(\boldsymbol{\omega}^{e}+\boldsymbol{\omega}^{em})$$
$$=\gamma_{1}\int_{V} dV(\boldsymbol{n}\times\partial_{t}\boldsymbol{n})-\int_{V} dV(\boldsymbol{\omega}^{e}+\boldsymbol{\omega}^{em}) \tag{12}$$

where we used the fact that when I is neglected we have $\boldsymbol{\tau}^{v}=\boldsymbol{n}\times\boldsymbol{h}^{v}$ $=\gamma_{1}(\boldsymbol{n}\times\partial_{t}\boldsymbol{n})$. The integral Eqs. (11) and (12) connect the total flux of intrinsic and orbital angular momentum entering the volume V with the change in time of $\boldsymbol{n}$. These equations show also that the intrinsic and the orbital angular momentum coming from outside are connected with $\boldsymbol{n}$ and its gradients, respectively.

All the experiments reported up till now were made using a homeotropically aligned nematic film of thickness L. The homeotropic alignment assures that the z-components of the surface integrals of the elastic fluxes $\hat{L}^{e}$ and $\hat{S}^{e}$ in Eqs. (11) and (12) are zero, as it is easily checked using Eqs. (4) and (7). The surface integrals of the z-components of the electromagnetic fluxes in Eqs. (11) and (12) reduce, in the paraxial approximation, to the differences ΔL_{z}^{em} and ΔS_{z}^{em} between the integrals $\iint L_{zz}^{em}dxdy$ and $\iint S_{zz}^{em}dxdy$ evaluated at the planes $z=0$ and $z=L$, with L_{zz}^{em} and S_{zz}^{em} given by Eqs. (9) and (10), respectively. These integrals parallel the quantum mechanical definitions of the average orbital and the spin angular momentum operators [15]. The internal torque $\boldsymbol{\tau}^{int}=\int_{V} dV(\boldsymbol{\omega}^{e}+\boldsymbol{\omega}^{em})$ on the right of Eqs. (11) and (12) is the $\hat{L}-\hat{S}$ coupling due to the elastic and optical anisotropy of the medium. When the stress tensors defined by Eqs. (4) and (5) are used, the elastic part of $\boldsymbol{\tau}^{int}$ vanishes in the one-elastic-constant approximation and the electromagnetic part of $\boldsymbol{\tau}^{int}$ vanishes in the small birefringence limit $\boldsymbol{\varepsilon}_{a}\rightarrow 0$. The internal torque $\boldsymbol{\tau}^{int}$ can be evaluated when an explicit form is given for the director field $\boldsymbol{n}(\boldsymbol{r},\ t)$. When an elliptically shaped Gaussian laser beam with different waists w_x and w_y in the x- and y-directions is used to reorient the liquid

crystal film, it is reasonable to assume that the optically induced distortion of $\boldsymbol{n}$ has an elongated Gaussian shape in the transverse plane. In general, however, the distribution of $\boldsymbol{n}$ may be elongated at some angle γ with respect to the x-axis. When the optical reorientation is small we may assume the following profile of $\boldsymbol{n}$

$$\begin{aligned} n_x &\cong A(t)\sin\left(\frac{\pi z}{L}\right)\cos\varphi(t)e^{-\frac{(x\cos\gamma+y\sin\gamma)^2}{a^2}-\frac{(y\cos\gamma-x\sin\gamma)^2}{b^2}} \\ n_y &\cong A(t)\sin\left(\frac{\pi z}{L}\right)\sin\varphi(t)e^{-\frac{(x\cos\gamma+y\sin\gamma)^2}{a^2}-\frac{(y\cos\gamma-x\sin\gamma)^2}{b^2}} \\ n_z &\cong 1 \end{aligned} \tag{13}$$

where $A(t)$ is a small quantity, $\varphi(t)$ is the azimuthal angle of $\boldsymbol{n}$, and a and b are the widths of the $\boldsymbol{n}$-distribution in the transverse plane (we assume $a > b$), and $\gamma = \gamma(t)$. The z-axis is chosen along the normal to the sample walls. Insertion of Eqs. (13) in the internal torque $\boldsymbol{\tau}^{int}$ yields $\boldsymbol{\tau}^{int}$ as a function of the parameters $A(t)$, $\varphi(t)$, and $\gamma(t)$. Insertion of Eqs. (13) into the viscous terms in Eqs. (11) and (12) yields the remarkable results

$$\begin{aligned} \gamma_1\int_V dV\partial_t n_\rho(\mathbf{r}\times\nabla)n_\rho &= -\frac{\gamma_1\pi L(a^2-b^2)^2A^2(t)}{16ab}\frac{d\gamma}{dt} \\ \gamma_1\int_V dV(\mathbf{n}\times\partial_t\mathbf{n}) &= \frac{1}{4}\gamma_1\pi LabA^2(t)\frac{d\varphi}{dt}. \end{aligned} \tag{14}$$

We see therefore that Eq. (12), which states the balance of the intrinsic angular momentum, yields an equation for the time derivative of the azimuthal angle $\varphi(t)$ of $\boldsymbol{n}$ as found long time ago in the plane wave approximation [7,8]. On the other hand, Eq. (11), which states the balance of the orbital angular momentum, yields an equation for the time derivative of the angle $\gamma(t)$, defining the orientation of the space distribution of $\boldsymbol{n}$ in the transverse plane. The angular momenta $\hat{L}$ and $\hat{S}$ act, therefore, on different degrees of freedom of $\boldsymbol{n}$. In particular, the orbital angular momentum $\hat{L}$ may play a role only if the space distribution of $\boldsymbol{n}$ is not uniform, where some angle γ can be defined. We used, indeed, Eqs. (11) and (12) and other integral relationships derived from the properties of the total free energy functional Φ to obtain a set of ordinary differential equations for $A(t)$, $\varphi(t)$, and $\gamma(t)$, to model the reorientation process, obtaining a fair agreement with the experimental findings [5].

4. CONCLUSIONS

We conclude by noticing that the free energy functional Φ is all we actually need in order to model the optically induced reorientation in liquid crystals

even when the orbital angular momentum of light is involved, whenever the motion of the fluid is completely neglected. In this case, an alternative possible approach is inserting the *ansatz* (13) in the free energy Φ and in the dissipation functional $R = \frac{\gamma_1}{2}\int dV(\partial_t \boldsymbol{n})^2$, and using the evolution equations $\frac{\partial \Phi}{\partial \gamma_i} = \frac{\partial R}{\partial \dot{\gamma}_i}$, where γ_i ($i = 1,2,\ldots,N$) are N time dependent parameters appearing in the distribution of $\boldsymbol{n}(\boldsymbol{r}, t)$. At equilibrium, we have $\frac{\partial \Phi}{\partial \gamma_i} = 0$ and the equilibrium values of the parameters γ_i are determined by Ritz variational principle. This approach yields a very good agreement with the observations made using linearly polarized light [16], thus proving that the fluid motion has a little effect, at least in the considered case, as shown in Figure 2. A more detailed work on the Ritz variational approach and its comparison with our experimental findings was presented elsewhere [16]. Accounting for the fluid motion and consequent backflow effects is a much more difficult problem and it will be postponed to further investigations.

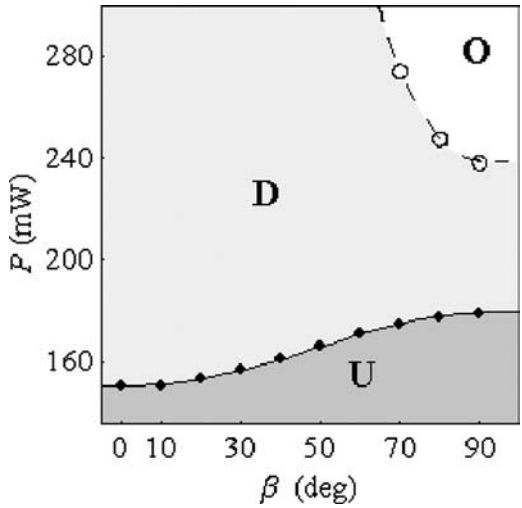

FIGURE 2 Map of the dynamical regimes observed varying the power P of an elliptically shaped laser beam at normal incidence. The beam polarization was linear at angle β with respect to the major axis of the intensity profile. Unlike in the traditional OFT with circularly shaped beams, nonlinear oscillating dynamical regimes were observed. Three regions may be recognized: **U**, undistorted states; **D**, steady distorted states; **O**, oscillating states. The continuous curve represents the threshold for the OFT and the dashed curve represents the threshold for the oscillations start up as calculated from our model. Full circles on the first curve and open circle on the second are the experimental threshold points.

REFERENCES

[1] Marrucci, L., Vetrano, F., & Santamato, E. (1999). *Opt. Commun.*, *171*, 131.
[2] Piccirillo, B. *et al.* (2001). *Phys. Rev. Lett.*, *86*, 2285.
[3] Piccirillo, B., Toscano, C., & Santamato, E. (2002). *Mol. Cryst. Liq. Cryst.*, *372*, 383.
[4] Santamato, E., Piccirillo, B., & Vella, A. (2002). *Mol. Cryst. Liq. Cryst.*, *375*, 601.
[5] Piccirillo, B., Vella, A., & Santamato, E. (2002). *J. Opt. B: Quantum Semicl. Opt.*, *4*, S20.
[6] Vella, A., Setaro, A., Piccirillo, B., & Santamato, E. (2003). *Phys. Rev. E*, *67*, 051704.
[7] Santamato, E. *et al.* (1986). *Phys. Rev. Lett.*, *57*, 2423.
[8] Santamato, *et al.* (1988). *Phys. Rev. Lett.*, *61*, 113.
[9] Galstyan, T. V. & Drnoyan, V. (1997). *Phys. Rev. Lett.*, *78*, 2760.
[10] De Gennes, P. G. (1974). The physics of liquid crystals. Clarendon Press: Oxford, UK.
[11] Gordon, J. P. (1973). *Phys. Rev. A*, *8*, 14.
[12] Barnett, S. M. (2002). *J. Opt. B: Quantum Semicl. Opt.*, *4*, S7.
[13] Forster, D. *et al.* (1971). *Phys. Rev. Lett.*, *26*, 1016.
[14] Landau, L. D. & Lifchitz, E. M. (1959). Electrodynamics of continuos media. Pergamon: London.
[15] van Enk, S. J. & Nienhuis, G. (1992). *Opt. Commun.*, *94*, 147.
[16] Piccirillo, B., Vella, A., Setaro, A., & Santamato, E. submitted to Phys. Rev. E.